Time Delays in the Synchronization of Chaotic Coupled Systems with Feedback

Jhon F. Martinez Avila and J. R. Rios Leite
Departamento de Física, Universidade Federal de Pernambuco, 50670-901, Recife, PE, Brazil

(Dated: March 22, 2009)

Isochrony and time leadership was studied in the synchronized excitable behavior of coupled chaotic lasers. Each unit of the system had chaos due to feedback with a fixed delay time. The inter-units coupling signal had a second, independent, characteristic time. Synchronized excitable spikes present isochronous, time leading or time lagging behavior whose stability is shown to depend on a simple relation between the feedback and the coupling times. Experiments on the synchronized low frequency fluctuations of two optically coupled semiconductor lasers and numerical calculations with coupled laser equations verify the predicted stability conditions for synchronization. Synchronism with intermittent time leadership exchange was also observed and characterized.

PACS numbers: 05.45.Xt,42.65.Sf,42.60.Mi

Many dynamical systems in nature have chaos due to feedback. A characteristic time, τ_F, associated to the feedback is therefore imbedded in the system response. As two or more of such systems are coupled, another independent time, τ_C, corresponding to the time of flight of the coupling signal, enters in the dynamical description of the global system. We present here how the relation between these two times determines the possible time delays in chaos synchronization. New features of time leadership competition in synchronized chaos between coupled pairs of systems with feedback are found when feedback and inter-coupling times have the same order of magnitude. Experimental and numerically, the stability of isochronous chaos synchronism for identical systems occurs only for specific relations between these times. We also show that time delayed and time advanced synchronism as well as synchronism with intermittent leadership exchange are also quantitatively determined by the ratio of these times. Our case is made with pairs of semiconductor diode lasers. However, the properties of synchronization in complex systems extends far beyond physical devices [1], reaching the subject of neural sciences [2].

The study of synchronism with chaotic lasers spreads for more than a decade [3, 4, 5, 6, 7]. Of relevant interest for applications are the results on the synchronization of semiconductor lasers [8, 9] for encrypted communication. With optical feedback diode lasers can present chaos in the form of very fast output power fluctuations, at a time scale of picoseconds. Superimposed on these, irregular power drops occur in a much slower time scale (order of hundreds of nanoseconds and longer) corresponding to the so called low frequency fluctuations (LFF) [10]. The LFF drops in single lasers are a current subject of studies and have been associated to spikes of excitable systems [11, 12, 13]. Coupled diode lasers show time advanced and time lagging synchronization via unidirectional coupling in master-slave configuration [14, 15] and in mutually coupled systems [16]. Symmetrically coupled pairs of lasers, without feedback, were shown to have unstable isochronous chaotic pulsation [16, 17, 18]. In the experiments and calculations with coupled lasers without feedback, the time leading lasers always appears with its power drops displaced by one unit of the coupling time, τ_C. The use of intermediate relaying system was demonstrated to give stable isochrony [19, 20]. Isochronous synchronization has also been investigated [20, 21, 22] for lasers with feedback with the studies focused on the fast laser fluctuations. Differently, herein we study the synchronization of the low frequency fluctuations (LFF), known to appear at the scale of hundreds of nanoseconds and slower. Thus, in our case the dynamics in the coupled systems is much faster than the synchronized events which have the properties of excitable spikes. So, our results refer to synchronism of excitable dynamical systems. The schema of the experiments is given in figure 1. Optical feedback was created in each laser by a retroreflecting external mirror. Their feedback return times, τ_{F1} and τ_{F2}, were set equal to within 1% precision (both named from hereon τ_F). The time of travel of the coupling signal between the lasers, τ_C, was independently controlled with respect to τ_F. Small changes in either of the times, on the scale of fraction of nanoseconds do not alter the properties of the synchronism. Thus the results,

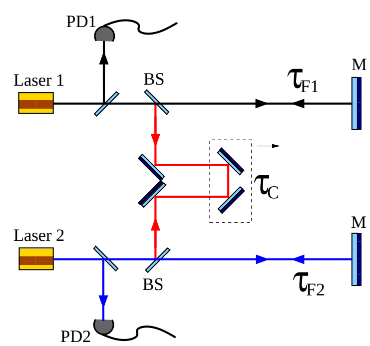

FIG. 1: Experimental setup for the synchronization of Low Frequency Fluctuations in optically coupled laser

like the LFF phenomenon in single laser with feedback, are robust with respect to optical phase changes. A consequence of such behavior is that our observations are consistent with the synchronization of excitable systems [23]. These authors show how unidirectionally coupled excitable systems can presents one of the systems always with a lower threshold for excitability. Their coupled systems synchronize to an external common signal. In our case the coupled systems have fast fluctuations and so, no external source of excitation is necessary.

The experiments were done with two SDL 5401 $GaAlAs$ semiconductor lasers, named here as Laser 1 and Laser 2, both with solitary threshold current of 21 mA and emitting at 805 nm. They were thermally stabilized to 0.01 K and could be temperature tuned to have their optical frequencies within 2 Ghz separation. The feedback time of the lasers was $\tau_F = 10\,ns$ and the strength were measured by the threshold reduction parameter, ξ which is the percentual variation of the threshold pump current as we consider the laser with and without feedback [10]. Symmetrical optical coupling between the lasers was produced with 50% beam splitters as shown in figure 1. The time of flight of the light between the lasers was varied between 5 and 20 ns. The threshold reductions due to cross input power were used to quantify the coupling strengths whose contribution to our studies will be detailed elsewhere. Manipulating the laser currents and temperatures, LFF synchronism was obtained where to each power drop in laser 1 corresponds a drop in laser 2 and vice versa. Typical experimental segments of the power output of the two lasers, with three events of the synchronized low frequency fluctuations (LFF) drops, are shown in figure 2. Each laser intensity was detected by a

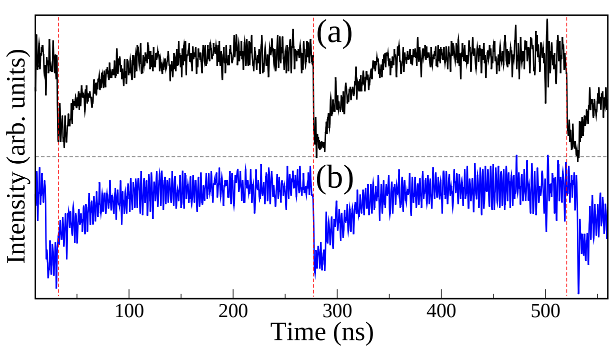

FIG. 2: Segment of experimental time series of the power of (a) laser 1 and (b) laser 2, showing the low frequency synchronism and including the interchange of time delay between the lasers. $\tau_F = 10$ ns and $\tau_C = 10$ ns.

1.5 GHz bandwidth photodiode. Simultaneous data series were acquired with a two channel digital oscilloscope having a bandwidth of 1 GHz and a maximum sampling rate of 5 GSamples/second. One of the lasers could always be the time leader by setting its pump current or its optical frequency [16] higher. However, as careful tuning of the laser currents and optical frequencies is made, within the same long LFF synchronized time series the LFF drops of laser 1 can appear isochronous, leading or lagging these of laser 2. Such were the conditions of figure 2 where the time leadership is interchanged between drops one and three, while the second LFF drops are isochronous. Using long pairs of data series, coarse grained to a $1\,ns$ time resolution [13], we measured $\Delta T_i = T_{1i} - T_{2i}$, the delay between the LFF drop i of laser 1 and laser 2.

The main result of this work is to show that, within any LFF synchronized evolution, for any pair of excited drops the allowed delays are given by

$$\Delta T_i = m_i \cdot \tau_C - n_i \cdot \tau_F \qquad (1)$$

where $m_i \neq 0$ and n_i are small integers. Here we only give indications of $m_i = \pm 1$ but preliminary results with very long numerical series show rare events with $m_i > 1$. Equation 1 covers various previously studied cases in the literature and confirms novel observations. For instance, with the lasers having no feedback, $\tau_F = 0$, it verified that isochrony is not allowed [16]. Other cases with feedback but forbidding isochrony are given below. The equation determines ΔT_i when one laser is always leading or lagging which happens for a single pair (m_i, n_i) through the whole time series. Furthermore, it is also valid for cases with varying values of (m_i, n_i) within the same series, obtained from symmetrical and nearly symmetrical systems. The LFF synchronism then appears with intermittent switching of the time delay that can interchange the leading subsystem. The evidence of equation 1 as a property of the LFF synchronism was obtained inspecting many different feedback and coupling delay times in experimental and numerical series. The origin of equation 1 is present in the intensity cross-correlation functions of the fast fluctuations of coupled lasers. These correlations, obtained with sub-nanosecond resolution, show recurrent narrow spikes (width of hundred of picoseconds) at positions and intervals given by 1.

Let us proceed presenting our experiments along with the numerical-theoretical results extracted from a system of differential equations that describe two monomode lasers with optical feedback and optical coupling. The model correspond to a set of modified Lang and Kobayashi [24] equations for the lasers with feedback, including symmetrical optical coupling and assumed to have the same optical frequency:

$$\begin{aligned} \frac{dE_i(t)}{dt} &= \frac{(1+i\alpha)}{2}\left[G(N_i) - \frac{1}{\tau_p}\right]E_i(t) + \kappa E_i(t-\tau_F) \\ &\quad +\gamma E_{j\neq i}(t-\tau_C) \qquad (2) \\ \frac{dN_i(t)}{dt} &= J_i - \frac{N_i(t)}{\tau_s} - G(N_i)\left|E_i(t)\right|^2, \qquad (3) \end{aligned}$$

where $i, j = 1, 2$ and each laser gain is given by

$$G(N_i) = \frac{G_0(N_i - N_0)}{1 + \epsilon\,|E_i(t)|^2} \qquad (4)$$

The definition of the various parameters and their typical values are well discussed in the literature [10, 12]. For each laser, E_j is the radiation field, α is the factor describing amplitude to phase conversion, G is the amplifying gain, $N_j(t)$ the carriers inversion population, τ_p the photon lifetime of the internal laser cavity, τ_s the carriers lifetime, J_j the threshold normalized pump currents and N_0 the inversion population for medium transparency. Each feedback field has an amplitude coefficient κ and feedback time τ_F. The optical couplings are linearly additive E field with coefficient γ and time τ_C for the field of one laser to reach the other one. Physical causality demands that both τ_F and τ_C be positive.

With a fourth order Runge-Kutta algorithm numerical data series were obtained for $E_i(t)$ and these gave the normalized intensity series $|E_i(t)|^2$. The time scales in the integrations and the equations parameters were fixed as: dt=1 ps, $1/\tau_p$=282 ns^{-1}, $N_0 = 1.5\times10^8$, $\epsilon = 5\times10^{-7}$, κ=$\gamma = 22$ ns^{-1}, $1/\tau_s = 1.66$ ns^{-1}. The times, τ_F and τ_C, are in the nanosecond range. Calculation with one laser having sufficiently higher pump current gives synchronized LFF with the laser of higher pump current always leading in time, as observed in the experiments. Equal pump currents ($J_i = 1.013$), corresponding to symmetrical systems, produced numerical time series with leadership exchange, again in agreement with the experiment. Segments of these data series are shown in figure 3, to be compared with the experimental segments in figure 2. Let us emphasize that no stochastic terms are used in

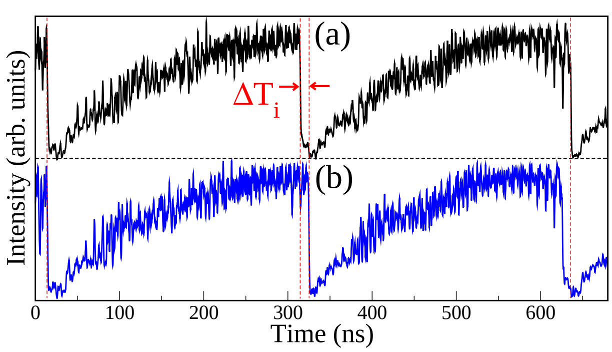

FIG. 3: Typical segment of the numerical time series of (a) laser 1 and (b) laser 2 LFF power drops under synchronized condition. Notice that the first drops are isochronous while time leadership was interchanged between drops two and three. $\tau_F = 10$ ns and $\tau_C = 10.05$ ns.

the equations. The apparent random time distribution of the LFF events in each laser as well as the time leadership switching in the synchronism is due to the high dimension deterministic chaos.

Time series with $\sim 10^4$ pairs of drops were coarse grain filtered [13] with a 1 ns time constant, and used to extract histograms of time delays between the drops. The switching of leadership from symmetrical systems was also examined and no correlation was found between consecutive values of ΔT_i, up to second order conditional probability. This is indication of a Markov process, obtained despite the fact that the data came from deterministic numerical equations with recurrence times τ_F and τ_C. Such result is consistent with high dimensional chaos with largely different time scales ($T_{i+1} - T_i \gg \tau_F, \tau_C$) that makes LFF appear as stochastic without memory [13]. The experimental data show the same lack of correlation for the delay times of LFF synchronism in symmetrical conditions. The histograms associated to the data series of figures 2 and 3, are shown in figure 4. As $\tau_F = 10$ ns and $\tau_C = 10$ ns, equation 1 with $m_i = \pm1$ and $n_i = \pm1$, predicts $\Delta T_i = 0, \pm10\,ns$ and $\pm20\,ns$. Indeed, the histograms show that isochrony occurs, along with time leadership exchange events. The major probabili-

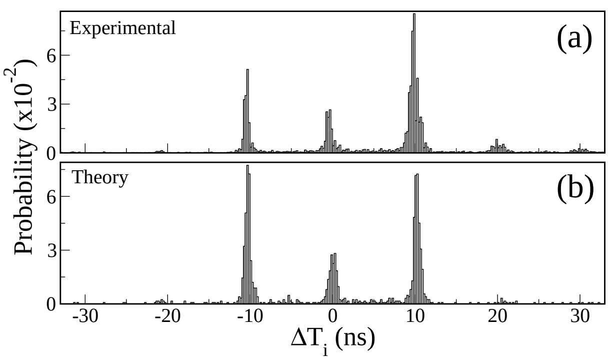

FIG. 4: Histograms of the (a) experimental and (b) numerical time delays between LFF pulses of the two synchronized lasers. $\tau_F = 10$ ns and $\tau_C = 10$ ns.

ties are for events with $\Delta T_i = 0$ and $\pm$ 10 ns, with few events at $\pm$20 ns and some in $\pm$30 on the experimental data. It is important to mention that on the calculations small differences (less than 2%) were introduced between the parameters of laser 1 and 2 in the equations. In particular $\tau_C \neq \tau_F$ ($\tau_C = 10.05$ ns) was necessary to give the non isochronous events. In fact such differences which are intrinsically present in the experiments change dramatically the amplitude of the peaks in the histograms but have minor effect on the values of allowed delays. The robustness of the relation between delay times and the condition of LFF synchronization according to equation 1 was also inspected for small, up to 10%, of relative variations of τ_F and τ_C.

Another typical numerical histogram where m_i and n_i change within the same synchronized evolution is shown in figure 5. This is also a case of symmetrical system where the laser parameters are equal but the values for time of coupling and feedback, were chosen to give unsta-

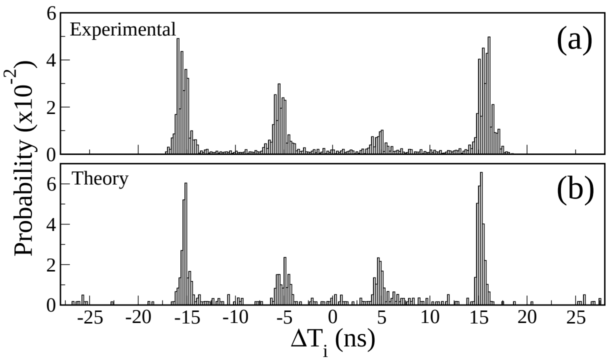

FIG. 5: Histograms from experimental (a) and numerical (b) series for the time delays between LFF drops of the two synchronized lasers: $\tau_F = 10$ ns and $\tau_C = 15$ ns.

ble isochrony: $\tau_C = 15$ ns and $\tau_F = 10$ ns. Accordingly, equation 1 these values prevent $\Delta T_{LFF} = 0$, as seen by direct substitution of small integers($m_i = \pm 1$). Thus, synchronism of the LFF holds, but there is always a finite time lead between the two lasers. The leadership can be exchange but events of simultaneous drops are excluded.

When $\tau_C \gg \tau_F$ we verified numerically that equation 1 still holds. The value of m_i is always ± 1 while n_i assumes a large range of values. Isochrony is absent and the dominant events occur with $n_i = 0$, corresponding to delays of $\pm\tau_C$. Nonsymmetrical systems, as mentioned above, also follow equation 1. A calculation with the two lasers having the same current $J_i = 1.013$ was given figure 6 (a) while figure 6 (b) shows what happens when Laser 1 has its current augmented to $J_1 = 1.025$. The laser with higher current is always the time leader, even though sometimes its leading time jumps down from τ_C to $\tau_C - \tau_F$. These events correspond to be $m_i = 1$ and $n_i = 0$ changing to $m_i = 1$ and $n_i = +1$ along the dynamical evolution with synchronized LFF excitations.

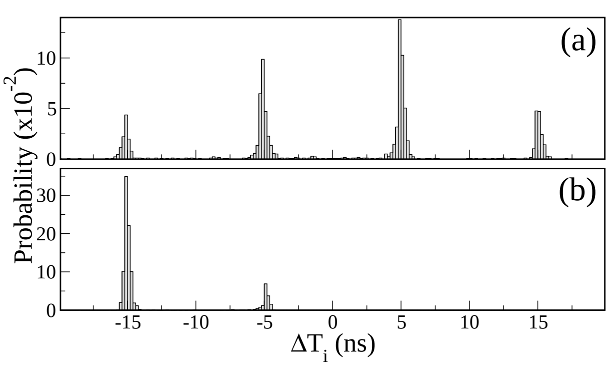

FIG. 6: Histograms of the numerical delay times between synchronized LFF drops of the lasers with $\tau_F = 10$ ns and $\tau_C = 15$ ns and showing the effect of asymmetric conditions. In (a). $J_i = 1.013$ for the two lasers and in (b) $J_1 = 1.025$ and $J_2 = 1.013$, making Laser 1 the time leader.

To summarize, from experiments with coupled lasers with feedback, corroborated by the numerical solutions of the corresponding rate equations, we discovered that the delay time in the synchronization of coupled excitable systems with feedback is controlled by a simple relation between feedback time and inter-coupling time. One of the systems may have a fixed time leadership or have its leading time switching values including isochrony. Nearly symmetrical systems can also have intermittent time leadership exchange always maintaining the excitable spikes synchronized in the large time scale. Specifically, the behavior of identical systems with feedback and coupled with time delay strongly depends on the relation between these times. A simple equation that specify the intercombinations of feedback and coupling time to give the allowed values of delay in the synchronized dynamics was introduced. Coupled asymmetrical systems also follow the conditions for the allowed delay time between events. The various parameters of the systems strongly influence the probability of specific delays, distorting the histogram amplitudes, but have no effect on the values of the time delays. Our results extend the previous [16] determination of the symmetry breaking and instability of expected isochrony in the synchronism of identical coupled chaotic systems without feedback. It also adds to the recently studied properties of isochronal synchronism done by Klein et al. [21] and Kanter et al. [7], who emphasize the potential application of these properties for encrypted communication. A formal mathematical treatment of the stable and unstable manifolds in chaotic synchronization with delays having integer combinations between feedback and coupling time will be presented elsewhere. The phenomenon of selection condition in time delays is bound to appear generically in mutually coupled dynamical systems.

The authors acknowledge useful discussions with H. L. D. de S. Cavalcante. Work partially supported by Brazilian Agencies Conselho Nacional de Pesquisa e Desenvolvimento (CNPq) and Fundação de Ciência de Pernambuco (FACEPE) and by a Brazil-France Capes-Cofecub project 456/04.

[1] A. Pikovsky, M. Rosenblum, and J. Kurths, *Synchronization: A Universal Concept in Nonlinear Sciences* (Cambridge Nonlinear Science Series, 2003).

[2] R. Vicente, L. L. Gollo, C. R. Mirasso, I. Fisher, and G. Pipa, PNAS- Proceedings of the National Academy of Sciences of the USA **105**, 17157 (2008).

[3] T. Sugawara, M. Tachikawa, T. Tsukamoto, and T. Y. Shimizu, Physical Review Letters **72**, 3502 (1994).

[4] R. Roy and K. S. Thornburg, Physical Review Letters **72**, 2009 (1994).

[5] Y. Liu, P. de Oliveira, M. Danailov, and J. R. Rios Leite, Physical Review A **50**, 3464 (1994).

[6] R. Meucci, F. Salvadori, M. V. Ivanchenko, K. A.

Naimee, C. Zhou, F. T. Arecchi, S. Boccaletti, and J. Kurths, Physical Review E **74**, 066207 (2006).

[7] I. Kanter, N. Gross, E. Klein, E. Kopelowitz, P. Yoskovits, L. Khaykovich, W. Kinzel, and M. Rosenbluh, Physical Reviews Letters **98**, 154101 (2007).

[8] Y. Takiguchi, H. Fujino, and J. Ohtsubo, Optics Letters **24**, 1570 (1999).

[9] A. Argyris, D. Syvridis, L. Larger, V. Annovazzi-Lodi, P. Colet, I. Fischer, J. Garcia-Ojalvo, C. R. Mirasso, L. Pesquera, and K. A. Shore, Nature **438**, 343 (2005).

[10] G. H. M. van Tartwijk and D. Lenstra, Quantum and Semiclassical Optics **7**, 87 (1995).

[11] M. Giudici, C. Green, G. Giacomelli, U. Nespolo, and J. R. Tredicce, Phys. Rev. E **55**, 6414 (1997).

[12] J. F. M. Avila, H. L. D. de S. Cavalcante, and J. R. Rios Leite, Physical Review Letters **93**, 144101 (2004).

[13] J. F. M. Avila, H. L. D. de S. Cavalcante, and J. R. Rios Leite, Physical Reviews Letters **100**, 044101 (2008).

[14] C. Masoller, Physical Review Letters **86**, 2782 (2001).

[15] S. Sivaprakasam, E. Shahverdiev, P. Spencer, and K. Shore, Physical Review Letters **87**, 154101 (2001).

[16] T. Heil, I. Fischer, W. Elsässer, J. Mulet, and C. R. Mirasso, Physical Review Letters **86**, 795 (2001).

[17] J. Mulet, C. Mirasso, T. Heil, and I. Fischer, Journal of Optics B: Quantum and Semiclassical Optics **6**, 97 (2004).

[18] E. A. Rogers-Dakin, J. García-Ojalvo, D. J. DeShazer, and R. Roy, Physical Review E **73**, 045201 (2006).

[19] I. Fischer, R. Vicente, J. M. Buldú, M. Peil, and C. R. Mirasso, Physical Review Letters **97**, 123902 (2006).

[20] I. B. Schwartz and L. B. Shaw, Physical Review E **75**, 046207 (2007).

[21] E. Klein, N. Gross, M. Rosenbluh, W. Kinzel, L. Khaykovich, and I. Kanter, Physical Review E **73**, 066214 (2006).

[22] N. Gross, W. Kinzel, I. Kanter, M. Rosenbluh, and L. Khaykovich, Optics Communication **264**, 464 (2006).

[23] M. Cisak, F. Marino, R. Toral, and S. Balle, Physical Review Letters **93**, 114102 (2004).

[24] R. Lang and K. Kobayashi, IEEE Journal of Quantum Electronics **QE-16**, 347 (1980).